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CONFLICT PLANNING FOR LOGICAL CONFLICTS IN RELATIVE FLIGHT OPERATIONS PLANNING

JULY 1966

L. Suyemoto

Prepared for
DEPUTY FOR ADVANCED PLANNING
DIRECTORATE OF SPECIAL SYSTEMS
ELECTRONIC SYSTEMS DIVISION
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L. G. Hanscom Field, Bedford, Massachusetts

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Project 6040
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ABSTRACT

Flight Operations Planning (FOP) is separated into relative FOP (planning) and exact FOP (scheduling). Conflict planning is the prescription of alternative possible time relations among activities whenever a conflict is met. The first part of this paper deals with conflict detection and the second part deals with conflict resolution in relative FOP.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

GENE D. MUNSON
Major, USAF

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SECTION I

INTRODUCTION

For a problem in Flight Operations Planning (FOP), a set of activities is set forth and a time relation is assigned to each activity of the set, either with respect to another activity or to a time line. As a result, two types of FOP are needed: relative FOP (planning) and exact FOP (scheduling). The separation of FOP into these two types is analogous to the separation of the planning and scheduling functions made by Kelly and Walker.[1]

In exact FOP, the time relation of each activity is precisely specified on a time line with a specified fixed origin. This means that the start and finish times of each activity are known with respect to the fixed origin of the time line, or can be determined from the given duration time of the activity. Mathematically, this means that functions from the set of activities have been assigned to the time line (e.g., $P(x)$ and $Q(x)$, Section II).

In relative FOP,[1,2] the time relation is with respect to other activities; e.g., activity x starts before activity y. There is a time line implicit in this case also, but the origin is not fixed and the time relation exists only between activities and not with respect to a fixed origin. The result of relative FOP is an ordering of activities consistent with the explicit or implied relations between those activities.

Relative FOP should precede exact FOP. In relative FOP, logical conflicts or incompatibilities can easily arise among time relations of activities (such as activity x simultaneously preceding and following activity y). These conflicts must be detected[3] and resolved before exact FOP can be attempted with any hope of success. If the number of activities is small, relative FOP and its associated logical conflict detection and resolution is possible manually. If, however, the number of activities is large, manual relative-time ordering of the activities may take an inordinate amount of time without certainty that the ordering is conflict-free. Therefore, for problems involving a large number of activities, the relative FOP process must be automated. When a set of activities has been determined to be logically conflict-free, exact FOP can be attempted.

Conflict resolution will be easier if known alternate time relationships are available for substitution for a conflicting time relation. (This is a subfunction of the Planning Function.)[1,2] The assignment of alternatives to resolve a conflict is called conflict planning.

Whenever a logical conflict arises in relative FOP, conflict planning prescribes the possible alternate valid time relations existing between two activities. In Section II of this paper, the time relations are expressed as logical conjunctions or disjunctions of a set of eight fundamental time relations between the start and finish times of activities. Section III of this paper describes the alternative time relations possible whenever a logical conflict is met. Thus, whenever a logical conflict is detected in a given set of activities and relations, valid time relations can be determined by negation of those time relations that contribute to the conflict.

In this report, the basis for conflict planning consists of tables of time relations in terms of conjunctions and disjunctions. From a short table (16 elements) of binary products of the eight fundamental relations, and from tables of relations derived by logical conjunctions of the eight fundamental relations, a method is derived for determining the higher-order products of time relations. Using the tables and the method, all possible time relations between activities can be derived. Thus, the implication and truth tables in Reference 3 will be subtables of those that can be derived.

There is a distinction between logical conflicts that may arise in relative FOP and conflicts that can arise when exact FOP is attempted. Successful relative FOP will give as an output a conflict-free time-ordering of a set of activities. Exact FOP, however, may yet lead to logical conflicts. Incompatibilities between the start and finish times of activities may arise when the duration times of the activities are placed on the time line. In Reference 3, it is shown that a logical conflict can arise only if, for a finite number of relations $R_1, R_2, \ldots, R_n$, the expression $xR_1y \wedge yR_2z \wedge \ldots \wedge wR_nx$ is obtained; i.e., activity x is both a first member of the relation xR_1y and a second member of the relation, wR_nx. Such an expression is called a cycle, shown graphically for $n = 4$ in Figure 1.

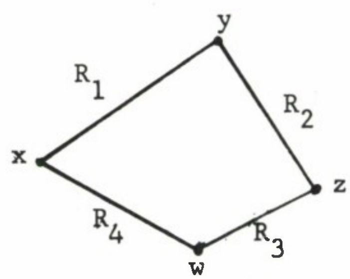
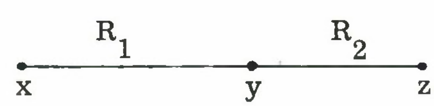


Figure 1. Graphic Presentation of a Cycle

For any other case, i.e., chains of activities and relations that contain no loops or cycles, the set of activities and time relations are logically conflict-free.

For exact FOP, this is not necessarily true. When exact FOP is attempted, even if a cycle does not exist and the relations and activities are of the form $xR_1y \wedge yR_2z$: graphically, this can be represented as



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the set of activities and associated time relations still may not be free of conflicts. An attempt to place the duration times on the time line, while keeping the ordering of activities fixed, can lead to conflicts. In this case, either the ordering must be changed to accommodate the duration time (i.e., alternate valid time relations must be substituted), or the duration times must be modified. The required modifications of the set of activities and relations and/or duration times will undoubtedly be dictated by the tasks to be performed. For example, the duration time for an activity may be minimal and, hence, the time relation needs to be altered; or the particular ordering of the activities may be mandatory, hence the duration times must be altered. The procedure for making the necessary modifications will be easier if the permissible valid time relations between activities are known.

Until a feasible exact schedule is determined, the complete processes of relative and exact FOP are, in general, iterative. It should be noted that preflight simulation of the different aspects of a mission may also necessitate repetition of relative FOP and exact FOP but with modified inputs. This can arise, for example, if the time alloted for an activity is found to be insufficient for completion of a task under simulation of the activity, or the particular sequencing of activities may not be feasible when placed on the time line prior to the simulation.

SECTION II

CONFLICT DETECTION

MATHEMATICAL DESCRIPTION

Relative Time Relations

Consider a set of activities X. Assume that, associated with each activity of this set, there is some description of time relative to at least one other activity in the set. This description could be either the relative start or finish time, or the relative time position of the duration of an activity with respect to another activity. A more precise definition of these relative time relations is required.

The relative time relations, overlap and nonoverlap, divide the set into all possible relative time relations; these two time relations are obviously exclusive. Given two activities, x and y, either x overlaps y or x does not overlap y, but both events cannot occur simultaneously. Moreover, the two relations are symmetric: x overlaps y is equivalent to y overlaps x, and x does not overlap y is equivalent to y does not overlap x.

A relation, R, can be defined as a set of ordered pairs. Mathematically, (x, y) is an element of the set if and only if xRy.[4] The two time relations, overlap and nonoverlap, operating on a given set of activities, X, generate two mutually exclusive sets of ordered pairs, i.e., two mutually exclusive relations. Subsets of these two mutually exclusive sets give all pairs of activities having explicit time relations between them. This is explained below.

The two basic time relations are too general and can lead to ambiguities, e.g., x overlaps y can mean that the start time of activity x occurs before the start time of activity y, or conversely. To remove the ambiguities, the symmetry property of the two relations is removed. In this paper, x laps y means that activity x starts before activity y and activity x overlaps activity y; x does not meet y means that activity x starts before activity y and activity x does not overlap activity y. Thus, the relation does-not-meet is equivalent to: either x does not meet y, or y does not meet x (but not both). The relation laps is equivalent to: either x laps y, or y laps x (but not both). These relations are shown diagrammatically in Figure 2.

Time Relation				Diagrammatically
(a)	x	does not meet	y	x y
(a')	y	does not meet	x	y x
(b)	x	laps	y	x y
(b')	y	laps	x	x y

Figure 2. Time Relations, Symmetry Removed

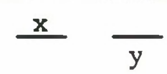
It should be noted that in all four statements in Figure 2, the relative start and finish times of x and y are implicit.

Using these relations, explicit time relations between two activities are easily obtained. Thus, x does not meet y means that x precedes y. A subset of the relation precedes is the set of ordered pairs where activity x immediately precedes y; i.e., the finish time of x is the same as the start time of activity y. These more explicit time relations are shown diagrammatically in Figures 3 through 7.

Henceforth, for ease in notation and writing, consideration of activities means the time relations between activities; e.g., x laps y means duration time of x overlaps the duration time of y, and activity x starts before activity y.

Introduction of the relative start and finish times between two activities gives all possible time relations that are more specific than the four time relations in Figure 2, as follows.

(a) x precedes y



(b) x immediately precedes y (finish time of x is the same as the start time of y)

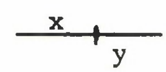


Figure 3. Time Relations from Figure 2 (a)

(a) y precedes x

(b) y immediately precedes x

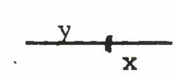
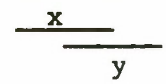
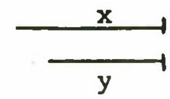


Figure 4. Time Relations from Figure 2 (a')

(a) x laps y



(b) x and y overlap and x and y have same finish times



(c) Activity y is contained in activity x

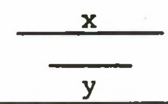
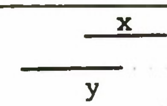
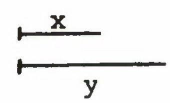


Figure 5. Time Relations from Figure 2 (b)

(a) y laps x



(b) y overlaps x and have the same start times



(c) Activity x is contained in activity y

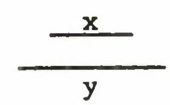


Figure 6. Time Relations from Figure 2 (b')

(a) x and y co-occur

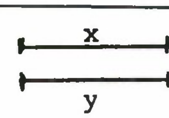


Figure 7. Combining Figures 5 (b) and 6 (b)

From the 11 relations described in Figures 3 through 7, all the relations described in Reference 3 can be derived.

Fundamental Set of Relations

In the time relations between activities described above, all the relative relationships of the start and finish times between activities x and y are implicitly understood. Thus, the relative start and finish times of an activity are considered as fundamental and a set of fundamental relations between two activities can be defined.

Consider a set of activities X, as the domain of two functions, P and and Q. The range of the two functions is a subset of the closed interval $[0, T]$ (the time line) on the real line. The two functions P and Q are defined as:

$$P(x) = t_0 \quad , \tag{1}$$

$$Q(x) = t_1 \quad , \tag{2}$$

where $t_0 < t_1$, and t_0 and t_1 can be thought of as the end points of an interval of the time line. Equation (1) is read: The start time of activity x is t_0. Equation (2) is read: The finish time of activity x is t_1. Equality between t_0 and t_1 can be admitted. The equality sign in $t_0 \leq t_1$ means that activity x has zero-duration time, but still may be used as a dummy variable in a net of activities to signify the direction of flow.

Given two activities, x and y ϵ x, there are only eight possibilities of the relationships of P(x), Q(x) with P(y), Q(y). (Obviously, it is true that $P(x) < Q(x)$ and $P(y) < Q(y)$, and, also obviously, it is false that $Q(x) \leq P(x)$ and $Q(y) \leq P(y)$, if all activities must have positive time durations.)

$$
\begin{array}{llll}
x\alpha y: & P(x) \leq P(y) & x\alpha' y: & P(y) \leq P(x) \\
x\beta y: & Q(x) \leq Q(y) & x\beta' y: & Q(y) \leq Q(x) \\
x\gamma y: & P(x) \leq Q(y) & x\gamma' y: & P(y) \leq Q(x) \\
x\delta y: & Q(x) \leq P(y) & x\delta' y: & Q(y) \leq P(x)
\end{array} \tag{3}
$$

From the fundamental set of eight relations, α , β , γ , δ , α' , β' , γ' , δ' , the 11 time relations given in Figures 3 through 7 can be derived. These are given in tables in the following section.

TABLES

Notation and Subcategories of the Eleven Relations to be Derived

Eleven relations, plus subcategories of these eleven relations, are listed below. It can be seen from Tables I through VI that these relations and subcategories are the only ones that arise from the logical conjunctions of the fundamental relations. The relations a_1 through a_{11} , and a'_7 a'_8 are called closed time relations, because the relative start and finish times of two activities are part of the relations. The remaining relations or subcategories of the relations are called open, since the start and/or finish times of either activity x or activity y , or both, are not part of the relation.

The open relations are listed because they arise naturally from the logical conjunctions of the fundamental relations. Furthermore, they could be useful when exact FOP is attempted; i.e., when duration times are considered. The open relations indicate that an activity is free to modify its start or finish time. For closed relations, two activities are fixed in relation to each other.

From the tables of logical conjunctions (Tables I through VI), it is evident that, taking 4-term, then 5-term,... 8-term conjunctions of distant fundamental relations leads to closed valid relations, or the conjunctions are incompatible (invalid). For example, for the 4-term logical conjunctions, all the valid relations are closed, or the 4-term logical conjunctions are invalid.

It is also evident from the tables of logical conjunctions that the time relations listed in Figures 8 through 18 can be expressed as conjunctions of fundamental relations, and conversely. Hence, these two sets of time relations are equivalent. The set a_1 through a_{11} contains all the Primitive Scheduling Constraints, Table 1, pg. 64, Reference 2.

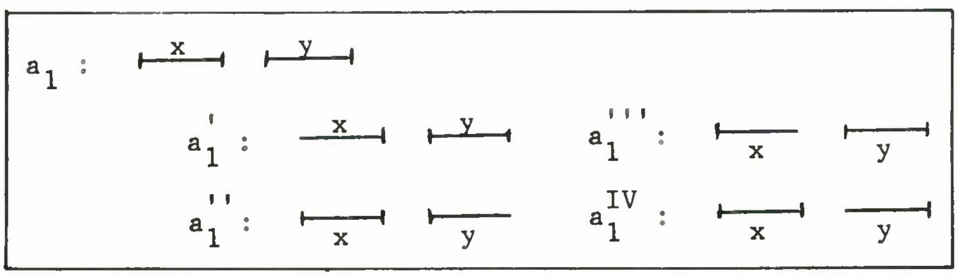


Figure 8. Time Relations, x Precedes y

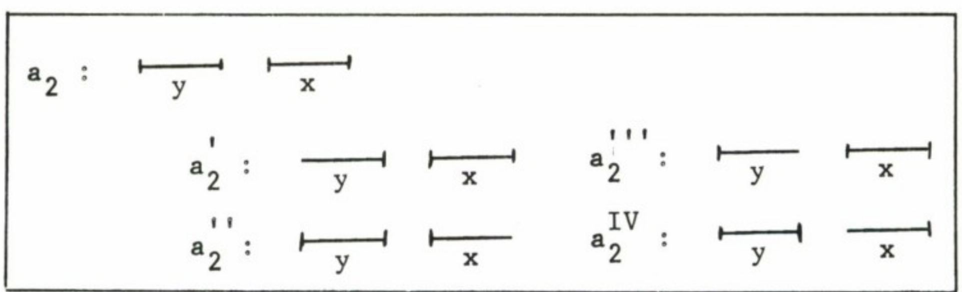


Figure 9. Time Relations, y Precedes x

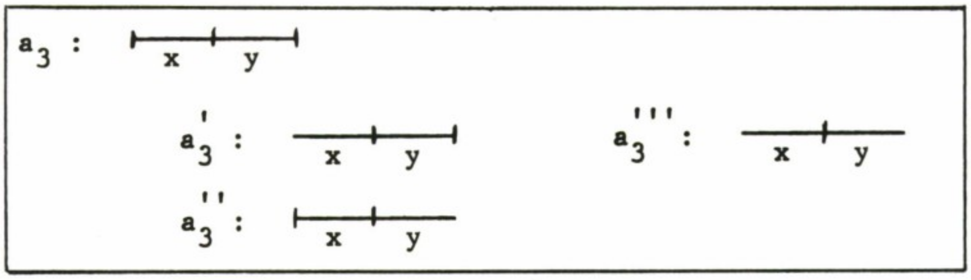


Figure 10. Time Relations, x Immediately Precedes y

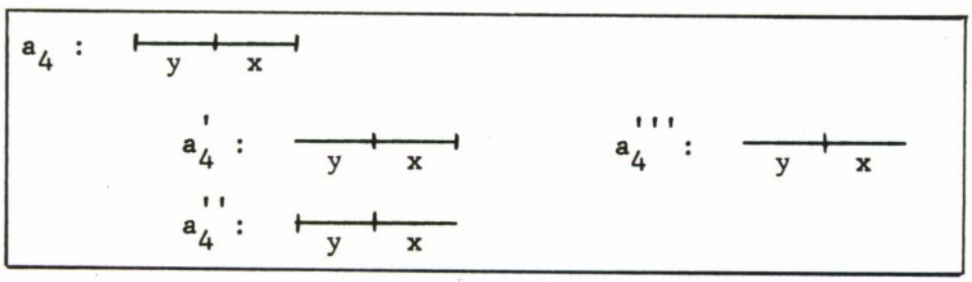


Figure 11. Time Relations, y Immediately Precedes x

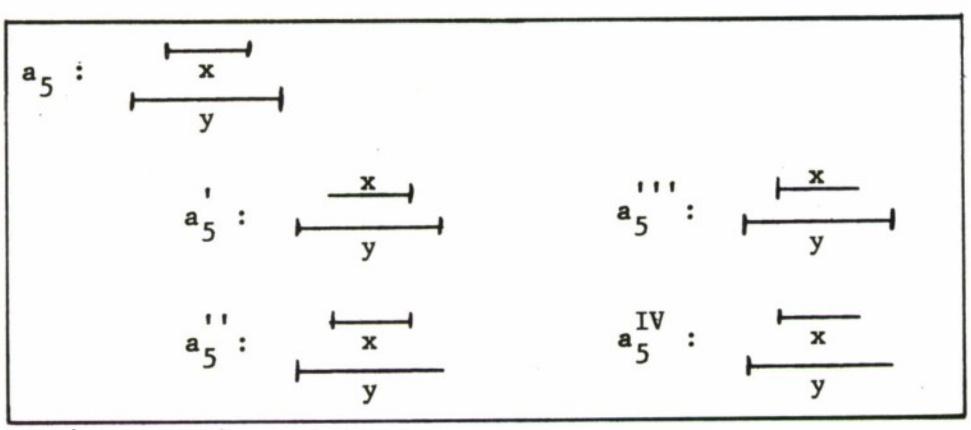


Figure 12. Time Relations, x Contained in y

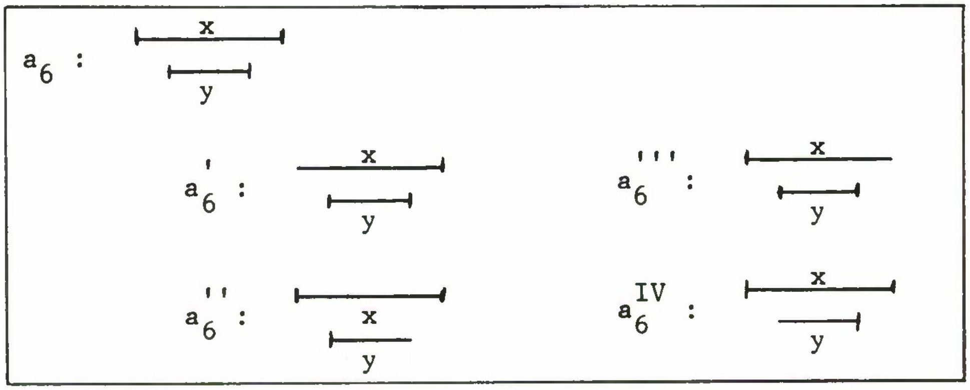


Figure 13. Time Relations, y Contained in x

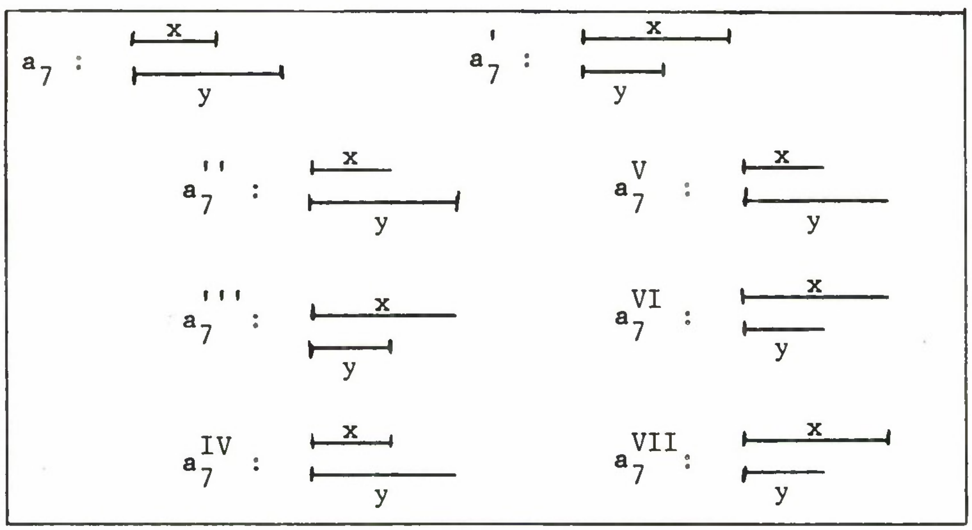


Figure 14. Time Relations, Same Start Times for x and y

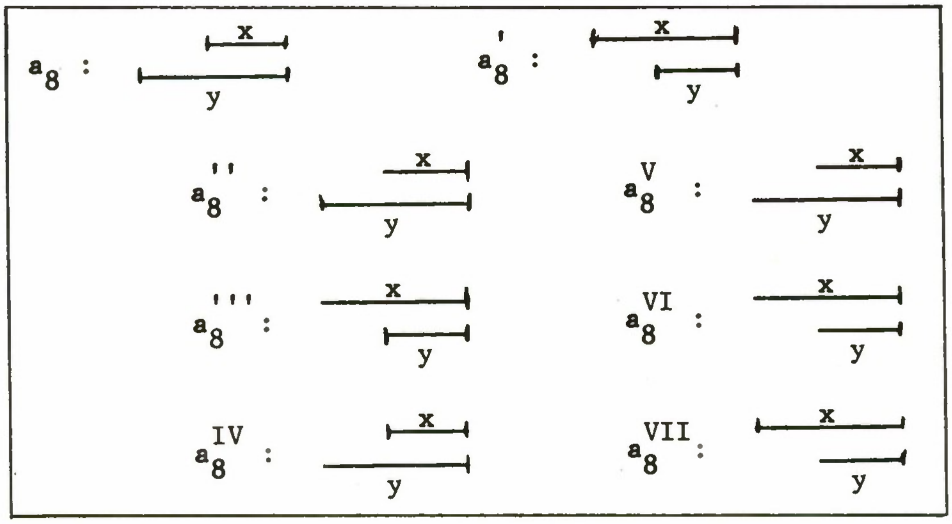


Figure 15. Time Relations, Same Finish Times for x and y

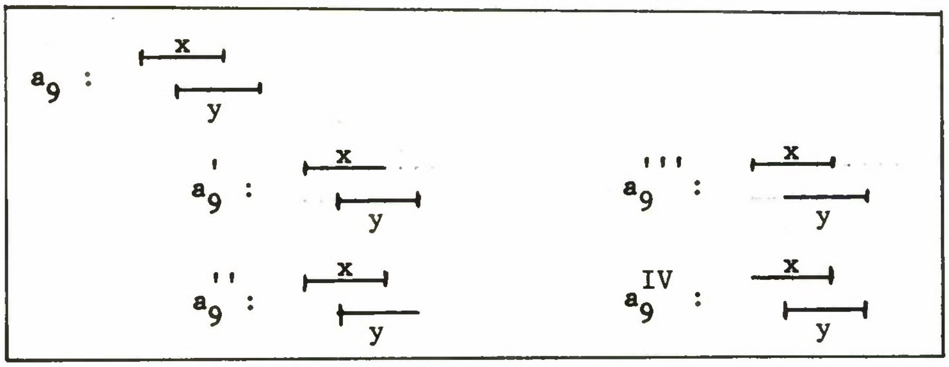


Figure 16. Time Relations, x Laps y

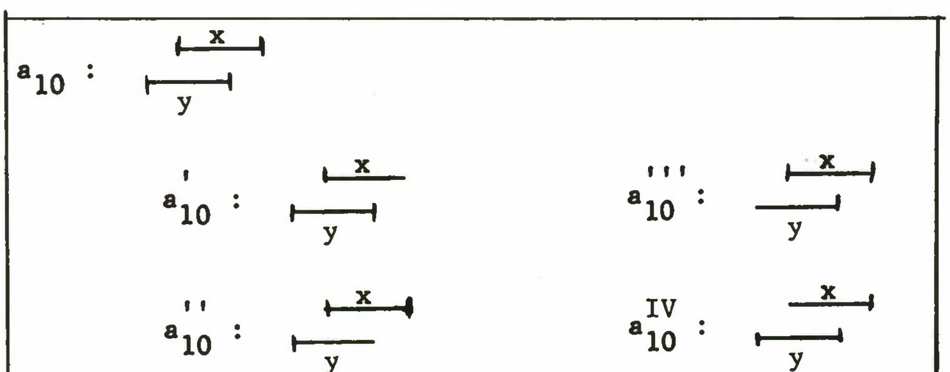


Figure 17. Time Relations, y Laps x

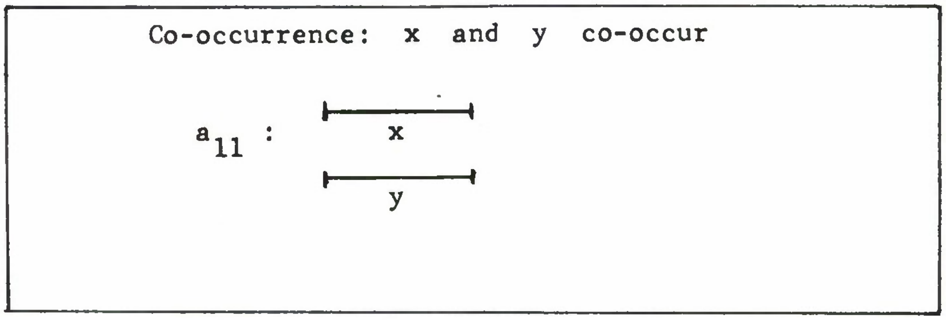


Figure 18. Time Relations, x and y Co-occur

Tables of Logical Conjunctions

The following tables are symmetric (the square arrays). Since logical conjunctions of time relations is a commutative operation, $\alpha \wedge \beta \equiv \beta \wedge \alpha$. The symbol X as an entry in the tables means that the conjunction is false, and the minus sign as an entry means (by commutativity) that the entry has already occurred elsewhere in the table.

Each entry in the table can be proved diagrammatically and/or by using the transitivity and idempotence property of logical conjunction. In Table I, three columns, α' , β' , δ' , have invalid conjunctions. Thus, logical conjunctions with six or more distinct fundamental relations are invalid.

The tables list all possible distinct logical conjunctions of the eight relations aside from the 6-, 7-, and 8-term conjunctions that are invalid. The tables consider the following ordering of the fundamental relations: α , β , γ , δ , α' , β' , γ' , δ' . For example, consider the logical conjunction of β' , γ , α'. This conjunction is considered in the form $\gamma \wedge \alpha' \wedge \beta'$. All other forms are equivalent since the relations are commutative with respect to logical conjunction. Thus, the tables do, in fact, give all possible distinct logical conjunctions. There are 218 of these distinct logical conjunctions (5-terms or less).

This total of 218 can be further reduced by considering the 36 distinct entries (valid and invalid) in Table I, and only the valid entries of the other tables. (The invalid relations in Table I are considered distinct since they are different combinations of the fundamental relations.) This can be done since all invalid entries in the tables of 3-, 4-, and 5-term logical conjunction have as factors $\alpha \wedge \delta'$, $\beta \wedge \delta'$, $\delta \wedge \beta'$, or $\delta \wedge \delta'$. There are 63 valid entries in Tables II through VI. Thus, a list of only 99 logical conjunctions or time relations needs to be considered.

PRODUCTS OF TWO FUNDAMENTAL RELATIONS

Definitions and Notations

The product of two relations, say R_1 and R_2 , over a set X, is defined in the following way: xR_1R_2z means that there exists a $y \in X$ such that xR_1y and yR_2z; or $(xR_1y \wedge R_2z)$.

Tables VII and VIII give the products of each pair of fundamental relations. The following notation is used for the entries. Disjunction, $\vee$, is defined as

$$\alpha \vee \beta = \alpha \text{ or } \beta \text{ or } \alpha \wedge \beta$$

$$\alpha \vee \beta \vee \gamma = \alpha \text{ or } \beta \text{ or } \gamma \text{ or } \alpha \wedge \beta \text{ or } \alpha \wedge \gamma \text{ or } \beta \wedge \gamma \text{ or } \alpha \wedge \beta \wedge \gamma \quad (4)$$

etc.

Table I

Two-Term Logical Conjunction Table

	α	β	γ	δ	α'	β'	γ'	δ'
α	α	$a_9 \vee a_1$	$a_9' \vee a_1''' \vee a_6'''$	a_1''	$a_7^{V} \vee a_7^{VI}$	a_6	$a_6'' \vee a_9''$	X
β	-	β	$a_9''' \vee a_1^{IV}$	a_1'	a_5	$a_8^{V} \vee a_8^{VI}$	$a_9^{IV} \vee a_5'$	X
γ	-	-	γ	a_1	$a_5''' \vee a_{10}^{IV}$	$a_6^{IV} \vee a_{10}'''$	$a_9 \vee a_5$	a_4'''
δ	-	-	-	δ	X	X	a_3'''	X
α'	-	-	-	-	α'	$a_{10} \vee a_{11}$	$a_{10}'' \vee a_5''$	a_2''
β'	-	-	-	-	-	β'	$a_6' \vee a_{10}^{IV}$	a_2'
γ'	-	-	-	-	-	-	γ'	a_2
δ'								δ'

Table II

Three-Term Logical Conjunction Table

	γ	δ	α'	β'	γ'	δ'
$\alpha\wedge\beta$	$a_1 \vee a_9$	a_1	a_7	a_8	a_9	X
$\alpha\wedge\gamma$	-	a_1	$a_7'' \vee a_7'''$	a_6	$a_9 \vee a_6$	X
$\alpha\wedge\delta$	-	-	X	X	a_3''	X
$\alpha\wedge\alpha'$	-	-	-	a_7'	$a_7^{VII} \vee a_7^{IV}$	X
$\alpha\wedge\beta'$	-	-	-	-	a_6	X
$\alpha\wedge\gamma'$	-	-	-	-	-	X
$\beta\wedge\gamma$		a_1	a_5	$a_8^{IV} \vee a_8^{VII}$	$a_9 \vee a_5$	X
$\beta\wedge\delta$		-	X	X	a_3'	X
$\beta\wedge\alpha'$		-	-	a_8	a_5	X
$\beta\wedge\beta'$		-	-	-	$a_8'' \vee a_8'''$	X
$\beta\wedge\gamma'$		-	-	-	-	X
$\gamma\wedge\delta$			X	X	a_3	X
$\gamma\wedge\alpha'$			-	a_{10}	$a_{10} \vee a_5$	a_4''
$\gamma\wedge\beta'$			-	-	$a_6 \vee a_{10}$	a_4'
$\gamma\wedge\gamma'$			-	-	-	a_4
$\delta\wedge\alpha'$				X	X	X
$\delta\wedge\beta'$				-	X	X
$\delta\wedge\gamma'$				-	-	X
$\alpha'\wedge\beta'$					$a_{10} \vee a_2$	a_2
$\alpha'\wedge\gamma'$					-	a_2
$\beta'\wedge\gamma'$						a_2

Table III

Four-Term Logical Conjunction Table A

	δ	α'	β'	γ'	δ'
$\alpha\wedge\beta\wedge\gamma$	a_1	a_7	a_8'	a_9	X
$\alpha\wedge\beta\wedge\delta$	-	X	X	a_3	X
$\alpha\wedge\beta\wedge\alpha'$	-	-	a_{11}	a_7	X
$\alpha\wedge\beta\wedge\beta'$	-	-	-	a_8'	X
$\alpha\wedge\beta\wedge\gamma'$	-	-	-	-	X
$\alpha\wedge\gamma\wedge\delta$		X	X	a_3	X
$\alpha\wedge\gamma\wedge\alpha'$		-	a_7'	$a_7\vee a_7'$	X
$\alpha\wedge\gamma\wedge\beta'$		-	-	a_6	X
$\alpha\wedge\gamma\wedge\gamma'$		-	-	-	X
$\alpha\wedge\delta\wedge\alpha'$			X	X	X
$\alpha\wedge\delta\wedge\beta'$			-	X	X
$\alpha\wedge\delta\wedge\gamma'$			-	-	X
$\alpha\wedge\alpha'\wedge\beta'$				a_7'	X
$\alpha\wedge\alpha'\wedge\gamma'$				-	X
$\alpha\wedge\beta'\wedge\gamma'$					X

Table IV

Four-Term Logical Conjunction Table B

	α'	β'	γ'	δ'
$\beta\wedge\gamma\wedge\delta$	X	X	a_3	X
$\beta\wedge\gamma\wedge\alpha'$	-	a_8	a_5	X
$\beta\wedge\gamma\wedge\beta'$	-	-	$a_8 \vee a_8'$	X
$\beta\wedge\gamma\wedge\gamma'$	-	-	-	X
$\beta\wedge\delta\wedge\alpha'$		X	X	X
$\beta\wedge\delta\wedge\beta'$		-	X	X
$\beta\wedge\delta\wedge\gamma'$		-	-	X
$\beta\wedge\alpha'\wedge\beta'$			a_8	X
$\beta\wedge\alpha'\wedge\gamma'$			-	X
$\beta\wedge\beta'\wedge\gamma'$				X
$\gamma\wedge\delta\wedge\alpha'$		X	X	X
$\gamma\wedge\delta\wedge\beta'$		-	X	X
$\gamma\wedge\delta\wedge\gamma'$		-	-	X
$\gamma\wedge\alpha'\wedge\beta'$			a_{10}	a_4
$\gamma\wedge\alpha'\wedge\gamma'$			-	a_4
$\gamma\wedge\beta'\wedge\gamma'$				a_4
$\delta\wedge\alpha'\wedge\beta'$			X	X
$\delta\wedge\alpha'\wedge\gamma'$			-	X
$\delta\wedge\beta'\wedge\gamma'$				X
$\alpha'\wedge\beta'\wedge\gamma'$				a_2

Table V

Five-Term Logical Conjunction Table A

	α'	β'	γ'	δ'
$\alpha\wedge\beta\wedge\gamma\wedge\delta$	X	X	a_3	X
$\alpha\wedge\beta\wedge\gamma\wedge\alpha'$	-	a_{11}	a_7	X
$\alpha\wedge\beta\wedge\gamma\wedge\beta'$	-	-	a_8'	X
$\alpha\wedge\beta\wedge\gamma\wedge\gamma'$	-	-	-	X
$\alpha\wedge\beta\wedge\delta\wedge\alpha'$		X	X	X
$\alpha\wedge\beta\wedge\delta\wedge\beta'$		-	X	X
$\alpha\wedge\beta\wedge\delta\wedge\gamma'$		-	-	X
$\alpha\wedge\beta\wedge\alpha'\wedge\beta'$			a_{11}	X
$\alpha\wedge\beta\wedge\alpha'\wedge\gamma'$			-	X
$\alpha\wedge\beta\wedge\beta'\wedge\gamma$				X
$\alpha\wedge\gamma\wedge\delta\wedge\alpha'$		X	X	X
$\alpha\wedge\gamma\wedge\delta\wedge\beta'$		-	X	X
$\alpha\wedge\gamma\wedge\delta\wedge\gamma'$		-	-	X
$\alpha\wedge\gamma\wedge\alpha'\wedge\beta'$			a_7'	X
$\alpha\wedge\gamma\wedge\alpha'\wedge\beta'$			-	X
$\alpha\wedge\gamma\wedge\beta'\wedge\gamma'$				X
$\alpha\wedge\delta\wedge\alpha'\wedge\beta'$			X	X
$\alpha\wedge\delta\wedge\alpha'\wedge\gamma'$			-	X
$\alpha\wedge\delta\wedge\beta'\wedge\gamma'$				X
$\alpha\wedge\alpha'\wedge\beta'\wedge\gamma'$				X

Table VI

Five-Term Logical Conjunction Table B

	β'	γ'	δ'
$\beta\wedge\gamma\wedge\delta\wedge\alpha'$	X	X	X
$\beta\wedge\gamma\wedge\delta\wedge\beta'$	-	X	X
$\beta\wedge\gamma\wedge\delta\wedge\gamma'$	-	-	X
$\beta\wedge\gamma\wedge\alpha'\wedge\beta'$		a_8	X
$\beta\wedge\gamma\wedge\alpha'\wedge\gamma'$		-	X
$\beta\wedge\gamma\wedge\beta'\wedge\gamma'$			X
$\beta\wedge\delta\wedge\alpha'\wedge\beta'$		X	X
$\beta\wedge\delta\wedge\alpha'\wedge\gamma'$		-	X
$\beta\wedge\delta\wedge\beta'\wedge\gamma'$			X
$\beta\wedge\alpha'\wedge\beta'\wedge\gamma'$			X
$\gamma\wedge\delta\wedge\alpha'\wedge\beta'$		X	X
$\gamma\wedge\delta\wedge\alpha'\wedge\beta'$		-	X
$\gamma\wedge\delta\wedge\beta'\wedge\gamma'$			X
$\gamma\wedge\alpha'\wedge\beta'\wedge\gamma'$			a_4
$\delta\wedge\alpha'\wedge\beta'\wedge\gamma'$			X

Table VII

Products of Two Fundamental Relations

	α	β	γ	δ	α'	β'	γ'	δ'
α	$\alpha \wedge (V \vee \beta' \vee \gamma')$	$\gamma \wedge (V \vee \alpha' \vee \beta' \vee \gamma')$	$\gamma \wedge (V \vee \alpha' \vee \beta' \vee \gamma')$	$\alpha \wedge (V \vee \beta' \vee \gamma')$	U	U	U	U
β	U	$\beta \wedge (V \vee \alpha' \vee \gamma')$	U	$\delta \wedge V$	U	U	U	U
γ	U	$\gamma \wedge (V \vee \alpha' \vee \beta' \vee \gamma')$	U	$\alpha \wedge (V \vee \beta' \vee \gamma')$	U	U	U	U
δ	$\delta \wedge V$	$\beta \wedge (V \vee \alpha' \vee \gamma')$	$\beta \wedge (V \vee \alpha' \vee \gamma')$	$\delta \wedge V$	U	U	U	U
α'	U	U	U	U	$\alpha' \wedge (\beta \vee \gamma \vee W)$	U	U	$\delta' \wedge W$
β'	U	U	U	U	$\gamma' \wedge (\alpha \vee \beta \vee \gamma \vee W)$	$\beta' \wedge (\alpha \vee \gamma \vee W)$	$\gamma' \wedge (\alpha \vee \beta \vee \gamma \vee W)$	$\beta' \wedge (\alpha \vee \gamma \vee W)$
γ'	U	U	U	U	$\gamma' \wedge (\alpha \vee \beta \vee \gamma \vee W)$	U	U	$\beta' \wedge (\alpha \vee \gamma \vee W)$
δ'	U	U	U	U	$\alpha' \wedge (\beta \vee \gamma \vee W)$	$\delta' \wedge W$	$\alpha' \wedge (\beta \vee \gamma \vee W)$	$\delta' \wedge W$

Table VIII

Products of Two Fundamental Relations (Reduced Table)

	α	β	γ	δ
α	$\alpha \wedge (V \vee \beta' \vee \alpha')$	$\gamma \wedge (V \vee \alpha' \vee \beta' \vee \gamma')$	$\gamma \wedge (V \vee \alpha' \vee \beta' \vee \gamma')$	$\alpha \wedge (V \vee \beta' \vee \gamma')$
β	U	$\beta \wedge (V \vee \alpha' \vee \gamma')$	U	$\delta \wedge V$
γ	U	$\gamma \wedge (V \vee \alpha' \vee \beta' \vee \gamma')$	U	$\alpha \wedge (V \vee \beta' \vee \gamma')$
δ	$\delta \wedge V$	$\beta \wedge (V \vee \alpha' \vee \gamma')$	$\beta \wedge (V \vee \alpha' \vee \gamma')$	$\delta \wedge V$

Tables of Products of Two Fundamental Relations

Let

$$U = \alpha \vee \beta \vee \gamma \vee \delta \vee \alpha' \vee \beta' \vee \gamma' \vee \delta'$$

$$V = \alpha \vee \beta \vee \gamma \vee \delta$$

$$W = \alpha' \vee \beta' \vee \gamma' \vee \delta' \qquad (5)$$

By the definition of operation $\vee$, U can be considered as the set of all possible logical conjunctions (including $\alpha \wedge \alpha = \alpha$, etc.), 2-, 3-, . . . , 8-term logical conjunctions of the fundamental relations. V and W also can be considered as the set of all possible logical conjunctions, 2-, 3-, and 4-term logical conjunctions of α, β, γ, δ, and α', β', γ', δ', respectively. However, in U, V, W, only valid conjunctions are considered. Thus, U can be viewed as the set of all valid time relations.

The tables are to be read row by column; e.g., second row by fourth column is read $\beta\delta$ and stands for $x \beta \delta z$.

Table VII can be simplified into a 4-by-4 table. Let R_1, R_2 be 2 relations. Define the operation prime, $'$, over conjunction, disjunction, and products, as follows.

$$(R_1 \wedge R_2)' = R_1' \wedge R_2'$$

$$(R_1 \vee R_2)' = R_1' \vee R_2'$$

$$(R_1')' = R_1$$

$$(R_1 R_2)' = R_2' R_1' \qquad \text{(order reversed)} \tag{6}$$

It readily follows that

$$\begin{aligned} U' &= U \\ V' &= W \\ W' &= V \end{aligned} \tag{7}$$

The reduced table is shown in Table VIII. Using the reduced table and the fact that the product of a primed and unprimed relation is U, and, by using the prime operation, all entries in Table VII are obtained.

METHOD FOR DERIVING HIGHER ORDER PRODUCTS OF RELATIONS

As stated previously, there are 63 valid time relations obtained from the logical conjunction of three or more fundamental relations, and 31 valid time relations from the 2-term logical conjunctions. Although some of the logical conjunctions are equivalent, e.g., $\alpha \wedge \beta \wedge \gamma = \gamma \wedge \delta = a_1$; if only the 31 valid two-term logical conjunctions are considered, there would be $(31)^2 - 64 = 897$ products to consider (excluding the 64 already given in Table VII), since the operation of product is not commutative. Thus, to get a complete list of all valid relations, a table of more than 1,000 entries would have to be made.

To avoid making such a large table, all the higher-order products of fundamental relations (e.g., $x\ \alpha\ \beta\ \gamma\ z$), and products of relations that are logical conjunctions of fundamental relations $\left(\text{e.g., } x(\gamma \wedge \alpha' \wedge \gamma')(\alpha \wedge \gamma)z\right)$, can be derived by use of the logical-conjunction tables and the table of binary products of fundamental relations, Table VII. The general procedure is to reduce the logical conjunction in the definition of products to logical conjunctions or disjunctions of 2-term logical conjunctions, by using the distributive laws of conjunctions and disjunctions. When reducing these logical conjunctions to the conjunctions or disjunctions of products of fundamental relations, use the commutativity and associativity of conjunctions and disjunctions, and also the idempotent law of conjunctions and disjunctions (where necessary). These general procedures can be better understood by the following examples.

<u>Example 1</u>

Consider the product $(\alpha \wedge \beta)\gamma$: $x(\alpha \wedge \beta)\gamma z$. By definition of product, there exists a y such that $x(\alpha \wedge \beta)y \wedge y\gamma z$

$\curvearrowright\ x\alpha y \wedge x\beta y \wedge y\gamma z$

$\curvearrowright\ [x\alpha y \wedge x\beta y \wedge y\gamma z] \wedge y\gamma z$

$\curvearrowright\ [x\alpha y \wedge y\gamma z] \wedge [x\beta y \wedge y\gamma z]$

$\curvearrowright\ [x\alpha\gamma z] \wedge [x\beta\gamma z]$

This proves that products are right-distributive over conjunctions, e.g.,

$$(\alpha\wedge\beta)\gamma \rightarrow \alpha\gamma\wedge\beta\gamma \tag{8}$$

It can be shown analogously that products are also left-distributive over conjunctions, e.g.,

$$\gamma(\alpha\wedge\beta) = \gamma\alpha\wedge\gamma\beta \tag{9}$$

The fact that products are both left- and right-distributive over conjunctions is useful for computational purposes.

From Table VII,

$$x\alpha\gamma z \;:\quad \alpha\wedge(\,V\vee\alpha'\vee\beta'\vee\gamma'\,)$$

$$x\beta\gamma z \;:\quad U$$

Thus, the product $x(\alpha\wedge\beta)\gamma z$ implies that activities x and z are related by the relation

$$\alpha\wedge[\,V\vee\alpha'\vee\beta'\vee\gamma'\,]\wedge U$$

This expression can be further reduced by the distributive laws of conjunctions and disjunctions.

<u>Example 2</u>

Consider the product $(\gamma\wedge\alpha'\wedge\gamma')(\alpha\wedge\gamma)$: $x(\gamma\wedge\alpha'\wedge\gamma')(\alpha\wedge\gamma)z$. There exists a y such that $x(\gamma\wedge\alpha'\wedge\gamma')\,y\wedge y\;(\alpha\wedge\gamma)z$

$$\rightarrow [x\gamma y\wedge x\alpha' y\wedge x\gamma' y]\wedge[y\alpha z\wedge y\gamma z]$$

$$\rightarrow [x\gamma y\wedge y\alpha z]\wedge[x\alpha' y\wedge y\gamma z]\wedge x\gamma' y$$

$$\rightarrow [x\gamma y\wedge y\alpha z]\wedge[x\alpha' y\wedge y\gamma z]\wedge[x\gamma' y\wedge y\gamma z]$$

$$\rightarrow [x\gamma\alpha z]\wedge[x\alpha'\gamma z]\wedge[x\gamma'\gamma z]$$

From Table VII,

$$x\gamma\alpha z \;:\; U$$

$$x\alpha'\gamma z \;:\; U$$

$$x\gamma'\gamma z \;:\; U$$

Thus, the product $x(\gamma \wedge \alpha' \wedge \gamma')\ (\alpha \wedge \gamma)z$ implies that x and z are related by the relation U.

Example 3

Consider the higher-order product $\alpha\beta\gamma \;:\; x\alpha\beta\gamma z$. By the associativity of products,

$$x\alpha\beta\gamma z = x\alpha(\beta\gamma)z = x(\alpha\beta)\gamma z$$

Consider $x\alpha(\beta\gamma)z$. Then there exists a y such that $x\alpha y \wedge y(\beta\gamma)z$. From Table VII, $\beta\gamma = U$. Thus, $y\beta\gamma z = yUz = y(\alpha \vee \beta \vee \ldots \vee \delta')z$.

$$x\alpha(\beta\gamma)z = x\alpha y \wedge yUz$$

$$x\alpha y \wedge yUz \rightarrow (x\alpha y) \wedge [(y\alpha z) \vee (y\beta z) \vee \cdots \vee (y\delta' z)]$$

$$\rightarrow [(x\alpha y) \wedge (y\alpha z)] \vee [(x\alpha y) \wedge (y\beta z)] \vee \ldots \vee [(x\alpha y) \wedge (y\delta' z)]$$

$$\rightarrow (x\alpha\alpha z) \vee (x\alpha\beta z) \vee \ldots \vee (x\alpha\delta' z)$$

From Table VII, the products $\alpha\alpha$, $\alpha\beta$, ..., $\alpha\delta'$ can be found.

Remarks

The method can be automated to generate the complete table of products, or as a subroutine to be used whenever it is required.

METHOD FOR DETERMINING THE VALIDITY OF PRODUCTS OF RELATIONS OF THE FORM xR_1R_2x

Reduction of Cycles to the Form xR_1R_2x

Logical conflicts may arise if an expression (cycle) of the form

$$xR_1y \wedge yR_2z \wedge \ldots \wedge wR_nx \tag{10}$$

is encountered. This chain of conjunctions can be reduced by the definition of products to an expression of the form

$$xR_1^*u \wedge uR_2^*x \quad . \tag{11}$$

This can be seen as follows.

$$xR_1y \wedge yR_2z = xR_1R_2z$$

$$(xR_1R_2z) \wedge zR_3v = x(R_1R_2)R_3v \tag{12}$$

$$\vdots$$

Finally, one arrives at a relation of the form $xR_1^*u \wedge uR_2^*x$.

The method for finding higher-order products reduces the products to the disjunctions of conjunctions of fundamental relations. Thus, if R_1^* and R_2^* in (11) are disjunctions of logical conjunctions of fundamental relations, the validity of (11) can be found from the truth or falsity of each of the logical conjunctions in R_1^* R_2^* by a table look-up of the tables of logical conjunctions.

Example 1

Let $R_1^* = \alpha$ and $R_2^* = \beta$ in expression (11). Consider the validity of the expression

$$x\alpha u \wedge u\beta x \tag{13}$$

From the fact that $u\beta x = x\beta' u$,

$$x\alpha u \wedge u\beta x = x\alpha u \wedge x\beta' u = x(\alpha \wedge \beta')u \tag{14}$$

From Table I, $\alpha \wedge \beta' = a_6$ and, hence, (13) is valid.

Example 2

Let $R_1^* = \alpha \vee \beta$ and $R_2^* = \gamma \wedge \delta$ in (11). Consider the validity of the expression

$$x(\alpha \vee \beta)u \wedge u(\gamma \wedge \delta)x \tag{15}$$

From the fact that $u(\gamma \wedge \delta)x = x(\gamma \wedge \delta)'u = x(\gamma' \wedge \delta')u$

$$\begin{aligned} x(\alpha \vee \beta)u \wedge u(\gamma \wedge \delta)x &= x(\alpha \vee \beta)u \wedge x(\gamma' \wedge \delta')u \\ &= x[(\alpha \wedge \gamma' \wedge \delta') \vee (\beta \wedge \gamma' \wedge \delta')]u \end{aligned} \tag{16}$$

From Table II, $\alpha \wedge \gamma' \wedge \delta'$ is false and $\beta \wedge \gamma' \wedge \delta'$ is false. Thus the expression in (15) is false.

SECTION III

CONFLICT RESOLUTION

Section II includes tables of logical conjunctions and binary products of fundamental relations, plus a method for obtaining higher-order products and products of logical conjunctions of fundamental relations. All the time relations that can exist between two activities are expressed in terms of logical conjunctions and disjunctions. A method is also given to determine whether a cycle (i.e., an expression of the form $xR_1y \wedge yR_2z \wedge \ldots \wedge wR_nx$, where $R_1, R_2 \ldots R_n$ are time relations, and x, y, z, ... are activities) does, indeed, have a logical conflict. Thus, the method, in conjunction with an algorithm for finding cycles, is a conflict-detection scheme.

In this Section (Section III), using what was developed in Section II, a method is given for obtaining all valid time relations between activities in a cycle that has a logical conflict. Logical conflicts arise if a cycle is invalid, i.e., the conjunctions of relations between activities are invalid. Because such cycles can be reduced to the form $xR_1^*u \wedge uR_2^*x$, by use of the definition of products of relations, a logical conflict essentially arises whenever it is false that $xR_1^*u \wedge uR_2^*x$. This means that, between two activities u and x, the two relations, R_1^* and R_2^*, are incompatible. The possible valid time relations can be determined from the negation of $xR_1^*u \wedge uR_2^*x$. A method for obtaining these valid time relations between activities x and u is given under Procedures for Conflict Planning (Section III).

For conflicts that arise when exact FOP is attempted (e.g., a string of conjunctions $xR_1y \wedge \ldots \wedge wR_nz$ that is logically free of conflicts may give rise to a logical conflict between the start and finish times of two activities when the duration times of the activities are included) the possible valid time relations can be obtained in an analogous way. Essentially, this type of conflict arises whenever it is false that xRy. The valid time relations can be found from the negation of R.

The process of obtaining alternative valid time relations for activities with a logical conflict is called conflict planning. Conflict planning can help the planners in the following ways. If the process of relative FOP has been automated, the computer can continue with its attempt to obtain a logically conflict-free time-ordering of the set of activities by using the valid alternate time relations. The end product of this operation will be a set of feasible time-orderings of the set of activities. Conflict planning will also help the planners by presenting to them all the valid possible time relations among the activities (essentially between two activities when the cycle is reduced) whenever the relations among them are invalid. The planners, with the knowledge of the task that needs to be performed (e.g., an experiment) can choose, perhaps by some priority scheme, the most appropriate valid time relation. (Some of the valid possible time relations may be inappropriate because of the physical and practical nature of the task to be performed.) In an extreme case, if none of the valid (logically) possible time relations are compatible with the nature of the task to be performed, the subset of activities given as input must be restructured.

As in Section II, equivalent ways of expressing time relations are given so that inputs (the set of activities and the time relations between activities)may be presented in various ways by the users. The set of time relations (used to express the relations between activities) to be given as input will depend on the user and also, perhaps, on the tasks to be performed. The criterion by which the quality of this set shall be judged will be the ease with which the particular set of time relations describes the required time relationships. In practice, the set of time relations used will probably be a mixture culled from different equivalent sets of time relations. For example, the fundamental set of relations may be, as a set, too general to use to prescribe the input. It may be easier to use a mixture of the fundamental time relations and the a_1 through a_{11} time relations. In any case, whatever time relations are used can be expressed in terms of the fundamental relations. Whatever computations are required will be made in terms of the fundamental relations.

OPEN AND CLOSED RELATIONS a_1 THROUGH a_{11} IN TERMS OF THE FUNDAMENTAL RELATIONS

Implications Existing Among the Fundamental Relations

From the definition of the fundamental relations, the following implications can be obtained, assuming all duration times are positive:

$$
\begin{array}{llll}
(a) & \alpha \rightarrow P(x) < Q(y) & (f) & \alpha' \rightarrow P(y) < Q(x) \\
(b) & \beta \rightarrow P(x) < Q(y) & (g) & \beta' \rightarrow P(y) < Q(x) \\
(c) & \delta \rightarrow P(x) < P(y) & (h) & \delta' \rightarrow P(y) < P(x) \\
(d) & \delta \rightarrow Q(x) < Q(y) & (i) & \delta' \rightarrow Q(y) < Q(x) \\
(e) & \delta \rightarrow P(x) < Q(y) & (j) & \delta' \rightarrow P(y) < Q(x)
\end{array}
\tag{17}
$$

These 10 implications can be proved by use of the transitivity property of logical conjunctions of relations and the fact that for an activity, x , $P(x) < Q(x)$ is always true. For example, proof of (a) in (17):

$$
\begin{aligned}
x\alpha y \equiv P(x) \leq P(y) &\rightarrow P(x) \leq P(y) \wedge P(y) < Q(y) \\
&\rightarrow P(x) < Q(y).
\end{aligned}
$$

From the list in (17), one would expect the following:

$$
\begin{array}{llll}
\text{(a)} & \alpha \wedge \gamma \rightarrow P(x) < Q(y) & \text{(f)} & \alpha' \wedge \gamma' \rightarrow P(y) < Q(x) \\
\text{(b)} & \beta \wedge \gamma \rightarrow P(x) < Q(y) & \text{(g)} & \beta' \wedge \gamma' \rightarrow P(y) < Q(x) \\
\text{(c)} & \delta \wedge \alpha \rightarrow P(x) < P(y) & \text{(h)} & \delta' \wedge \alpha' \rightarrow P(y) < P(x) \\
\text{(d)} & \delta \wedge \beta \rightarrow Q(x) < Q(y) & \text{(i)} & \delta' \wedge \beta' \rightarrow Q(y) < Q(x) \\
\text{(e)} & \delta \wedge \gamma \rightarrow P(x) < Q(y) & \text{(j)} & \delta' \wedge \gamma' \rightarrow P(y) < Q(x)
\end{array} \tag{18}
$$

Each of these 10 implications can be proved as above.

It should be noted that each of the implied relations (the strict inequality between start and finish times) is a fundamental relation with equality excluded. (If the duration times are assumed to be non-negative, i. e., zero-duration times are admitted, the implied relations will then be a fundamental relation.)

List of Relations

The following list of relations, a_1 through a_{11}, in terms of the fundamental relations, is obtained from the tables of logical conjunctions in Section II.

$$
\begin{array}{lll}
\text{(a)} & a_1 & = \gamma \wedge \delta \\
 & & = \alpha \wedge \beta \wedge \delta = \alpha \wedge \gamma \wedge \delta = \beta \wedge \gamma \wedge \delta \\
 & & = \alpha \wedge \beta \wedge \gamma \wedge \delta \\
 & a_1' & = \beta \wedge \delta \\
 & a_1'' & = \alpha \wedge \delta
\end{array} \tag{19}
$$

(b) $a_2 = \gamma' \wedge \delta'$

$$= \alpha' \wedge \beta' \wedge \delta' = \alpha' \wedge \gamma' \wedge \delta' = \beta' \wedge \gamma' \wedge \delta'$$

$$= \alpha' \wedge \beta' \wedge \gamma' \wedge \delta'$$

$a_2' = \beta' \wedge \delta'$

$a_2'' = \alpha' \wedge \delta'$

(c) $a_3 = \gamma \wedge \delta \wedge \gamma'$

$$= \alpha \wedge \beta \wedge \delta \wedge \gamma' = \alpha \wedge \gamma \wedge \delta \wedge \gamma' = \beta \wedge \gamma \wedge \delta \wedge \gamma'$$

$$= \alpha \wedge \beta \wedge \gamma \wedge \delta \wedge \gamma'$$

$a_3' = \beta \wedge \delta \wedge \gamma'$

$a_3'' = \alpha \wedge \delta \wedge \gamma'$

$a_3''' = \delta \wedge \gamma'$

(d) $a_4 = \gamma \wedge \gamma' \wedge \delta'$

$$= \gamma \wedge \alpha' \wedge \beta' \wedge \delta' = \gamma \wedge \alpha' \wedge \gamma' \wedge \delta' = \gamma \wedge \beta' \wedge \gamma' \wedge \delta'$$

$$= \gamma \wedge \alpha' \wedge \beta' \wedge \gamma' \wedge \delta'$$

$a_4' = \gamma \wedge \beta' \wedge \delta'$

$a_4'' = \gamma \wedge \alpha' \wedge \delta'$

$a_4''' = \gamma \wedge \delta'$ (19 con't)

(e) $a_5 = \beta \wedge \alpha'$

$= \beta \wedge \gamma \wedge \alpha' = \beta \wedge \alpha' \wedge \gamma'$

$= \beta \wedge \gamma \wedge \alpha' \wedge \gamma'$

(f) $a_6 = \alpha \wedge \beta'$

$= \alpha \wedge \gamma \wedge \beta' = \alpha \wedge \beta' \wedge \gamma'$

$= \alpha \wedge \gamma \wedge \beta' \wedge \gamma'$

(g) $a_7 = \alpha \wedge \beta \wedge \gamma \wedge \alpha' = \alpha \wedge \beta \wedge \alpha' \wedge \gamma'$

$= \alpha \wedge \beta \wedge \gamma \wedge \alpha' \wedge \gamma'$

$a_7' = \alpha \wedge \alpha' \wedge \beta'$

$= \alpha \wedge \gamma \wedge \alpha' \wedge \beta' = \alpha \wedge \alpha' \wedge \beta' \wedge \gamma'$

$= \alpha \wedge \gamma \wedge \alpha' \wedge \beta' \wedge \gamma'$

(h) $a_8 = \beta \wedge \alpha' \wedge \beta'$

$= \beta \wedge \gamma \wedge \alpha' \wedge \beta' = \beta \wedge \alpha' \wedge \beta' \wedge \gamma'$

$= \beta \wedge \gamma \wedge \alpha' \wedge \beta' \wedge \gamma'$

$a_8' = \alpha \wedge \beta \wedge \beta' \wedge \gamma' = \alpha \wedge \alpha' \wedge \beta' \wedge \gamma'$

$= \alpha \wedge \beta \wedge \gamma \wedge \beta' \wedge \gamma'$

(i) $a_9 = \alpha \wedge \beta \wedge \gamma'$

$= \alpha \wedge \beta \wedge \gamma \wedge \gamma'$

(j) $a_{10} = \gamma \wedge \alpha' \wedge \beta'$

$= \gamma \wedge \alpha' \wedge \beta' \wedge \gamma'$ (19 con't)

(k) $a_{11} = \alpha \wedge \beta \wedge \alpha' \wedge \beta'$

$$= \alpha \wedge \beta \wedge \gamma \wedge \alpha' \wedge \beta' = \alpha \wedge \beta \wedge \alpha' \wedge \beta' \wedge \gamma' \quad \text{(19 concl'd)}$$

Remarks

The fact that time relations a_1 through a_{11} have multiple representations with respect to the logical conjunctions of fundamental relations can be readily seen from the list of implication in (17) and (18).

The prime (') operation is merely an interchange of x and y with respect to the functions P and Q; e.g., $x \alpha y \equiv P(x) \leq P(y)$ and $x \alpha' y \equiv P(y) \leq P(x)$ (definition of the fundamental relations, Section II). Using the definition of the prime operation over logical conjunction and disjunction given in Section II, some similarity might be expected among the relations a_1 through a_{11}, in terms of the fundamental relations. This is easily seen between the pairs a_1 and a_2, a_3 and a_4, a_5 and a_6, a_9 and a_{10}. Thus, the prime operation reduces the number of time relations that must be listed. This could be useful if storage in a computer is of importance.

NEGATION OF TIME RELATIONS

Negation of Fundamental Time Relations

The notation $\overline{R}$ will be used for negation of the relation R. For example, the expression $x \overline{\alpha} y$ is read: it is false that $P(x) \leq P(y)$. If it _is_ false that $P(x) \leq P(y)$, then the only valid time relation between the start-time of activity x and the start-time of activity y is $P(y) < P(x)$ (strict inequality). Thus, from the definition of fundamental relations, negation of the eight fundamental relations is as follows:

$$
\begin{array}{llll}
(a) & \overline{\alpha} : P(y) < P(x) & (e) & \overline{\alpha'} : P(x) < P(y) \\
(b) & \overline{\beta} : Q(y) < Q(x) & (f) & \overline{\beta'} : Q(x) < Q(y) \\
(c) & \overline{\gamma} : Q(y) < P(x) & (g) & \overline{\gamma'} : Q(x) < P(y) \\
(d) & \overline{\delta} : P(y) < Q(x) & (h) & \overline{\delta'} : P(x) < Q(y)
\end{array} \tag{20}
$$

The conjunction of a fundamental relation and an appropriate primed fundamental relation will give a time relation expressing the equality of the start or finish times of two activities. For example, consider the conjunction of α and α'. By the definition of α and α', $P(x) \leq P(y)$, $P(y) \leq P(x)$. The equality between the start time of activities x and y is expressed by $x\alpha \wedge \alpha' y$, i.e.

$$
[P(x) \leq P(y) \wedge P(y) \leq P(x)] \equiv [P(x) = P(y)] \tag{21}
$$

Thus

$$
\begin{array}{llll}
(a) & \alpha \wedge \alpha' : P(x) = P(y) & (c) & \gamma \wedge \delta' : P(x) = Q(y) \\
(b) & \beta \wedge \beta' : Q(x) = Q(y) & (d) & \delta \wedge \gamma' : P(y) = Q(x)
\end{array} \tag{22}
$$

Using (22), (20) can be written as

$$
\begin{array}{llll}
(a) & \overline{\alpha} = \alpha' \wedge \overline{(\alpha \wedge \alpha')} & (e) & \overline{\alpha'} = \alpha \wedge \overline{(\alpha \wedge \alpha')} \\
(b) & \overline{\beta} = \beta' \wedge \overline{(\beta \wedge \beta')} & (f) & \overline{\beta'} = \beta \wedge \overline{(\beta \wedge \beta')} \\
(c) & \overline{\gamma} = \delta' \wedge \overline{(\gamma \wedge \delta')} & (g) & \overline{\gamma'} = \delta \wedge \overline{(\delta \wedge \gamma')} \\
(d) & \overline{\delta} = \gamma' \wedge \overline{(\delta \wedge \gamma')} & (h) & \overline{\delta'} = \gamma \wedge \overline{(\gamma \wedge \delta')}
\end{array} \tag{23}
$$

The eight negations in (20) and (23) relate to negation of a fundamental relation. This is in contrast to a situation where a time relation is invalid between two activities in a network of activities. Actually, all the valid time relations are required. Thus, in conflict planning, the invalid time relation found by the logical-conflict-detection scheme is negated to determine all the possible valid time relations. If the invalid relation between two activities is a fundamental time relation, the valid time relations are:

$$
\begin{array}{llll}
\text{(a)} & \overline{\alpha} : \overline{\alpha} \wedge U = \alpha' \wedge \overline{(\alpha \wedge \alpha')} \wedge U & \text{(e)} & \overline{\alpha'} : \overline{\alpha'} \wedge U = \alpha \wedge \overline{(\alpha \wedge \alpha')} \wedge U \\
\text{(b)} & \overline{\beta} : \overline{\beta} \wedge U = \beta' \wedge \overline{(\beta \wedge \beta')} \wedge U & \text{(f)} & \overline{\beta'} : \overline{\beta'} \wedge U = \beta \wedge \overline{(\beta \wedge \beta')} \wedge U \\
\text{(c)} & \overline{\gamma} : \overline{\gamma} \wedge U = \delta' \wedge \overline{(\gamma \wedge \delta')} \wedge U & \text{(g)} & \overline{\gamma'} : \overline{\gamma'} \wedge U = \delta \wedge \overline{(\delta \wedge \gamma')} \wedge U \\
\text{(d)} & \overline{\delta} : \overline{\delta} \wedge U = \gamma' \wedge \overline{(\delta \wedge \gamma')} \wedge U & \text{(h)} & \overline{\delta'} : \overline{\delta'} \wedge U = \gamma \wedge \overline{(\gamma \wedge \delta')} \wedge U
\end{array}
\tag{24}
$$

where U is as defined in Section II.

The term U can be considered as the set of all valid time relations (all the valid logical conjunctions in the tables of logical conjunctions). Thus, for example, $\overline{\alpha} \wedge U$ is the set of all valid time relations for which the logical conjunction of α', with an element of the set U, is valid, excluding all conjunctions with $\alpha \wedge \alpha'$ as a factor. Whether or not these logical conjunctions are valid can be determined by a table look-up in the tables of logical conjunctions.

Note that the implied relations in (17) and (18) can be expressed in terms of a negated fundamental relation. Thus, $\alpha \curvearrowright \overline{\delta'}$, $\alpha \curvearrowright \overline{\delta'}$, etc.

Negation of Two or More Term Logical Conjunctions of Fundamental Relations

The negation of a two-term logical conjunction, say α and β, is

$$\overline{\alpha \wedge \beta} = \overline{\alpha} \vee \overline{\beta} \tag{25}$$

where $\bar{\alpha} \vee \bar{\beta}$ means $\bar{\alpha}$ or $\bar{\beta}$ or $\bar{\alpha} \wedge \bar{\beta}$. (25)

If, for two activities, the relation $\alpha \wedge \beta$ is incompatible, then

$$\overline{\alpha \wedge \beta} = (\bar{\alpha} \wedge U) \vee (\bar{\beta} \wedge U) = (\bar{\alpha} \vee \bar{\beta}) \wedge U \quad (26)$$

The expression on the right side of (26) gives all the possible time relations (with redundant relations excluded) whenever $\alpha \wedge \beta$ is invalid.

The negations of 3-, 4-, or 5-term logical conjunctions follow analogously.

Negation of Derived Relations a_1 Through a_{11} in Terms of a_1 Through a_{11}

The list shown in Figure 19 gives the valid derived relations (including subcategories except where noted) whenever a derived relation is negated.

PROCEDURES FOR CONFLICT PLANNING

Two Cases Requiring Conflict Planning

As stated at the beginning of this Section, conflict planning can be applied in two instances. In the case of relative FOP, logical conflicts arise, essentially, when, for two relations, R_1 and R_2, and two activities, x and y, it is false that $xR_1y \wedge yR_2x$.

$$\overline{xR_1R_2x} \equiv \overline{xR_1y \wedge yR_2x} \quad (27)$$

Writing the negation as a disjunction,

$$\overline{xR_1y \wedge yR_2x} = \overline{xR_1y} \vee \overline{yR_2x} \quad (28)$$

Negated Relation	Valid Relations
$\bar{a}_1$	a_2 , a_4 - a_{11} .
$\bar{a}_2$	a_1 , a_3 , a_5 - a_{11} .
$\bar{a}_3$	a_1 , a_2 , a_4 - a_{11} .
$\bar{a}_4$	a_1 - a_3 , a_5 - a_{11} .
$\bar{a}_5$	a_1 - a_4 , a_6 , $a_7^{'}$, $(a_7^{'''}$, a_7^{VI} , $a_7^{VII})$, $a_8^{'}$, $(a_8^{'''}$, a_8^{VI} , $a_8^{VII})$, a_9 - a_{10} .
$\bar{a}_6$	a_1 - a_5 , a_7 , $(a_7^{''}$, a_7^{IV} , $a_7^{V})$, a_8 , $(a_8^{''}$, a_8^{IV} , $a_8^{V})$, a_9 - a_{10} .
$\bar{a}_7$	a_1 - a_6 , a_8 - a_{10} .
$\bar{a}_8$	a_1 - a_7 , a_9 , a_{10} .
$\bar{a}_9$	a_1 - a_6 , a_{10} .
$\bar{a}_{10}$	a_1 - a_6 , a_9 .
$\bar{a}_{11}$	a_1 - a_6 , a_9 , a_{10} .

Figure 19. Valid Relations for Negated Relations

For exact FOP, even if the set of activities under consideration is conflict-free, the duration times given for the activities may cause logical conflicts between activities when exact FOP is attempted. Thus, for this case, a conflict essentially arises if, for a relation, R, and two activities, x and y, it is false that xRy,

$$\overline{xRy} \quad . \tag{29}$$

Thus, in either (28) or (29), essentially it is only necessary to find the possible valid time relations for the negation of the expression xRy.

Procedures For Computation

Invalid Fundamental Relations

If the relation R is a fundamental invalid relation between two activities, the list given in (24) will give the possible valid time relations between the two activities. The explicit time relations can be found by a table look-up. For example, suppose that it is false that $x\alpha y$, i.e., $x\bar{\alpha}y$. Then, from the list given in (24), $\overline{x\alpha y}$ implies $x(\bar{\alpha} \wedge U)y$. The set of all explicit valid time relations can be found by looking down the α' column in the tables of logical conjunctions and across the α' row (excluding all conjunctions having $\alpha \wedge \alpha'$ as a factor); i.e., from the tables of logical conjunctions find all valid logical conjunctions that contain the term α' (excluding those including $\alpha \wedge \alpha'$ as a factor).

Invalid Logical Conjunction of Fundamental Relations

If the relation, R, is a logical conjunction of two fundamental relations that is invalid between two activities, x and y, then the procedure for finding the valid time relations between x and y is as follows. For example, suppose that it is false that $x\alpha \wedge \beta y$, i.e., $\overline{x\alpha \wedge \beta y}$. The valid time relations are given by

$$\overline{x\alpha \wedge \beta y} = x(\bar{\alpha} \vee \bar{\beta})y \ : \ x[(\bar{\alpha} \vee \bar{\beta}) \wedge U]y \tag{30}$$

$$= x(\bar{\alpha} \wedge U)y \vee x(\bar{\beta} \wedge U)y$$

The logical conjunctions have been reduced to the consideration of fundamental relations. By a table look-up, the explicit valid time relations for $\overline{\alpha} \wedge U$ and $\overline{\beta} \wedge U$ can be found. The sum of these two sets gives all of the valid time relations between activities x and y.

For relations that are logical conjunctions of 3-, 4-, or 5-fundamental relations, the procedure is analogous.

It should be noted that the negation of a product of two relations, R_1 and R_2, is reduced to the negation of logical conjunctions by the definition of products of relations.

$$\overline{xR_1R_2z} = \overline{(xR_1y) \wedge (yR_2z)} = \overline{xR_1y} \vee \overline{yR_2z} \tag{31}$$

Invalid Derived Time Relations a_1-a_{11}

If the relation, R, is a derived relation, say a_1, that is invalid between two activities x and y, then the procedure to find the valid time relations between x and y is as follows. For example, the valid time relations when $\overline{xa_1y}$, in terms of a_1 through a_{11}, can be found from Figure 19. The valid derived relations in terms of the fundamental relations can be found from the list given in (19), pages 34 through 37 of this report.

Example 1

Assume that $x\alpha \wedge \beta y$ is invalid.

$$\overline{x\alpha \wedge \beta y} = \overline{x\alpha y} \vee \overline{x\beta y} \tag{32}$$

From (24),

$$\overline{\alpha} : \alpha' \wedge \overline{(\alpha \wedge \alpha')} \wedge U \qquad \overline{\beta} : \beta' \wedge \overline{(\beta \wedge \beta')} \wedge U \tag{33}$$

From the tables of logical conjunctions (ommitting all redundancies) the list of valid time relations are as shown in Figure 20.

$\bar{\alpha} \wedge U$	$\bar{\beta} \wedge U$
$\beta \wedge \alpha' = a_5$	$\alpha \wedge \beta' = a_6$
$\gamma \wedge \alpha' = a_5''' \vee a_{10}^{IV}$	$\gamma \wedge \beta' = a_6^{IV} \vee a_{10}'''$
$\bar{\alpha} \wedge \alpha' = \bar{\alpha}$	$\bar{\beta} \wedge \beta' = \bar{\beta}$
$\alpha' \wedge \beta' = a_{10} \vee a_{11}$	$\beta' \wedge \gamma' = a_6' \vee a_{10}^{IV}$
$\alpha' \wedge \gamma' = a_{10}'' \vee a_5''$	$\beta' \wedge \delta' = a_2'$
$\alpha' \wedge \delta' = a_2''$	
$\beta \wedge \gamma \wedge \alpha' = a_5$	$\alpha \wedge \gamma \wedge \beta' = a_6$
$\beta \wedge \alpha' \wedge \gamma' = a_5$	$\alpha \wedge \beta' \wedge \gamma' = a_6$
$\gamma \wedge \alpha' \wedge \beta' = a_{10}$	$\gamma \wedge \beta' \wedge \gamma' = a_6 \vee a_{10}$
$\gamma \wedge \alpha' \wedge \gamma' = a_{10} \vee a_5$	$\gamma \wedge \beta' \wedge \delta' = a_4'$
$\gamma \wedge \alpha' \wedge \delta' = a_4''$	$\beta' \wedge \gamma' \wedge \delta' = a_2$
$\alpha' \wedge \beta' \wedge \gamma' = a_{10} \vee a_2$	
$\alpha' \wedge \beta' \wedge \delta' = a_2$	
$\gamma \wedge \alpha' \wedge \beta' \wedge \gamma' = a_{10}$	$\gamma \wedge \beta' \wedge \gamma' \wedge \delta' = a_4$
$\gamma \wedge \alpha' \wedge \beta' \wedge \delta' = a_4$	
$\gamma \wedge \alpha' \wedge \gamma' \wedge \delta' = a_4$	
$\alpha' \wedge \beta' \wedge \gamma' \wedge \delta' = a_2$	
$\gamma \wedge \alpha' \wedge \beta' \wedge \gamma' \wedge \delta' = a_4$	

Figure 20. List of Valid Time Relations

If the alternative time relations are to be considered in terms of the derived relations, the procedure is as follows. From the two-term logical conjunction table,

$$\alpha \wedge \beta = a_9 \vee a_1 \tag{34}$$

Then

$$\overline{\alpha \wedge \beta} = \overline{a_9 \vee a_1} = \bar{a}_9 \wedge \bar{a}_1 \tag{35}$$

From the list in Figure 19, the valid relations for $\bar{a}_9 \wedge \bar{a}_1$ are a_2, a_4 through a_6, and a_{10} (including the subcategories). The possible fundamental relations for the valid derived relations (i. e., possible variations of the start and finish times to derive the various derived relations), can be found in the list given in (19).

Remarks

(1) The list in Figure 20 gives all possible valid time relations. The list of logical conjunctions of fundamental relations, as determined from the derived relations using the list in (19), gives only the conjunctions for the open and closed derived relations (excluding the disjunction of derived relations). Thus, the latter list is a sublist of the former.

(2) The list of possible valid conjunctions of fundamental relations can be reduced by using the list of implications in (17) and the list of relations in (19).

(3) Even with reduced lists, the number of possible valid logical conjunctions that must be considered for conflict planning may be formidable (i. e., if many logical conflicts occur). This, however, is to be expected in any planning where the number of activities is large. If, for example, there are two logical conflicts in a set of activities, and for each logical conflict there are 10 possible valid relations, there can exist 10^2 feasible orderings for the set of activities. In the conventional method of planning,

the planners have repeated conferences to overcome this combinatorial problem (by essentially a trial-and-error process). For automated planning, there are certain possible alternatives:

(a) List all feasible orderings.

(b) If a complete listing of all feasible orderings is impractical, give the possible valid time relationships only when a conflict is met. Let the planner choose a valid time relation and then continue the ordering until another conflict is met. Repeat the procedure (man-machine interface).

(c) Establish prior rules, such as, whenever a conflict is met, the time relation chosen among the valid time relations is that closest to the invalid time relation. For example: if the relation a_1 is invalid (x precedes y), choose a_9 (x laps y) or if a_5 is invalid (x is contained in y), choose a_7 (x and y have the same start time).

Example 2

Consider the example of a logical conflict given in Reference 3, page 14. The two time relations used are inclusion, a_5, and same finish time, a_8. The inclusion relation used in the example is strict inclusion. Thus, the relations a_7 (same start time), a_8 (same finish time), and a_{11} (co-occurrence) are excluded as possible subcases of the relation of inclusion. The diagram of the relations is shown in Figure 21.

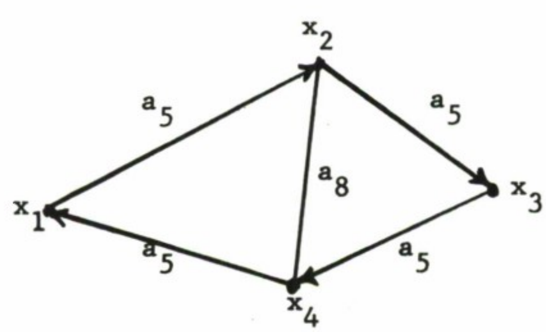


Figure 21. Diagram of Relations

Figure 21 is a directed network, and, since a_8 (same finish time) is a symmetric relation, there are three cycles in the diagram.

$$(x_1 a_5 x_2) \wedge (x_2 a_5 x_3) \wedge (x_3 a_5 x_4) \wedge (x_4 a_5 x_1) \tag{36}$$

$$(x_1 a_5 x_2) \wedge (x_2 a_8 x_4) \wedge (x_4 a_5 x_1) \tag{37}$$

$$(x_2 a_5 x_3) \wedge (x_3 a_5 x_4) \wedge (x_4 a_8 x_2) \tag{38}$$

Cycles (37) and (38) are valid [Ref. 3]. For cycle (36), it can be seen, by the transitivity property of relations and conjunction, that

$$(x_1 a_5 x_2) \wedge (x_2 a_5 x_3) \rightarrow x_1 a_5 x_3$$

$$(x_1 a_5 x_3) \wedge (x_3 a_5 x_4) \rightarrow x_1 a_5 x_4 \tag{39}$$

Finally,

$$(x_1 a_5 x_4) \wedge (x_4 a_5 x_1) \tag{40}$$

Because of the condition of strict inclusion, the last relation is invalid.

For conflict planning,

$$\overline{(x_1 a_5 x_4) \wedge (x_4 a_5 x_1)} = \overline{(x_1 a_5 x_4)} \vee \overline{(x_4 a_5 x_1)} \tag{41}$$

The valid derived relations are, from Figure 19.

$$\overline{x_1 a_5 x_4} : a_1 - a_4 , a_6 , a_7' , (a_7''' , a_7^{VI} , a_7^{VII}) ,$$
$$a_8' , (a_8''' , a_8^{VI} , a_8^{VII}) , a_9 , a_{10}$$

$$\overline{x_4 a_5 x_1} : a_1 - a_4 , a_6 , a_7' , (a_7''' , a_7^{VI} , a_7^{VII}) ,$$
$$a_8' , (a_8''' , a_8^{VI} , a_8^{VII}) , a_9 , a_{10} \tag{42}$$

In terms of the fundamental relations, consider, from (19) that

$$a_5 = \beta \wedge \alpha' \tag{43}$$

Then from (41)

$$\overline{(x_1 a_5 x_4) \wedge (x_4 a_5 x_1)} = \overline{[x_1(\beta \wedge \alpha')x_4] \wedge [x_4(\beta \wedge \alpha')x_1]}$$

$$= \overline{x_1(\beta \wedge \alpha')x_4} \vee \overline{x_4(\beta \wedge \alpha')x_1} \tag{44}$$

Thus,

$$x_1\overline{(\beta \wedge \alpha')}x_4 \ : \ \overline{\beta \wedge \alpha'} \rightarrow (\bar{\beta} \vee \overline{\alpha'}) \wedge U$$

$$x_4\overline{(\beta \wedge \alpha')}x_1 \ : \ \overline{\beta \wedge \alpha'} \rightarrow (\bar{\beta} \vee \overline{\alpha'}) \wedge U \tag{45}$$

Since, from the negation of fundamental relations β and α' , $x\alpha \wedge \alpha' y$ (same start time) and $x\beta \wedge \beta' y$ (same finish time) are invalid, the valid relations of $(\bar{\beta} \vee \overline{\alpha'}) \wedge U$ can be listed from the tables of logical conjunctions.

SUMMARY OF PROCEDURES FOR CONFLICT PLANNING

Summary of Procedures for Conflict Planning for Logical Conflicts in Relative FOP

(a) Given: an input comprising a set of activities and associated time relations. The time relations between two activities can be in terms of the fundamental relations [(3), Section II], derived relations (see under Tables, Section II), or a combination of the fundamental relations and the derived relations.

(b) Apply an algorithm to find all cycles of the form $(xR_1y) \wedge (yR_2z) \wedge \ldots \wedge (wR_nx)$.

(c) For each cycle found,reduce the cycle to a form $xRy \wedge yR^*x$ ($= xRR^*x$) by the reduction method described in Section II (Reduction of Cycles to the Form xR_1R_2x).

(d) The validity of $xRy \wedge yR^*x$ can also be determined by the method used for determining the validity of products of relations of the form xR_1R_2x.

(e) If there is a logical conflict, the existing valid time relations between x and y for $\overline{xRy \wedge yR^*x}$ can be found by using the method described under Procedures for Conflict Planning, Section III.

<u>Summary of Procedures for Conflict Planning When Exact FOP is Attempted</u>

(a) Given: a set of logically conflict-free activities on which relative FOP has been performed.

(b) Given: an input comprising duration times of the activities.

(c) Attempt exact FOP.

(d) For two activities, x and y, and relation R, xRy may be invalid (e.g., the duration time between the activities invalidates the relative-time relation between the start and finish times of activities x and y.)

(e) The existing valid time relations for $\overline{xRy}$ can be found by using the method given under Procedures for Conflict Planning, Section III.

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13. ABSTRACT

Flight Operations Planning (FOP) is separated into relative FOP (planning) and exact FOP (scheduling). Conflict planning is the prescription of alternative possible time relations among activities whenever a conflict is met. The first part of this paper deals with conflict detection and the second part deals with conflict resolution in relative FOP.

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It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS), (S), (C), or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, snd weights is optional.